INTERCELL AND INTRACELL COLLISION RESOLUTION IN WIRELESS PACKET DATA NETWORKS.

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Abstract - We address a new medium access algorithm that allows many wireless terminals to access a network of base stations on a cellular grid. Our *Frame Algorithm* **allows extremely dense frequency reuse as it not only resolves message collisions within a cell but also controls mutual interference among adjacent cells. We compute throughput and packet delays considering path loss and Rayleigh-fading.**

1 Introduction

This paper examines an access method that efficiently handles bursty multimedia traffic in wireless networks with multiple base stations. We propose a method that combines dynamic frequency reuse and random access. This is in contrast to existing cellular systems that typically separate these two issues. Concerning random access within one cell, the popular ALOHA scheme has the advantage of its simplicity. However, for an infinite population of users, it is unstable and the number of previously unsuccessful packets grows beyond any finite bound. To avoid this, collision resolution algorithms have been proposed. The early algorithms were sensitive to errors in the feedback information if the base station makes an error in judging what happened in a slot. In a wireless channel, packets may be lost because of signal fading even if no contending other signal is present. On the other hand, packets may be received successfully despite interference from competing terminals, which is called `receiver capture'. This present paper studies an improved version of the Frame Algorithm, which was proposed in [1, 2]. It is very robust against feedback errors. Moreover, it does not need a distinction between `success' (in the absence of competing transmissions) and `capture' (with competition). These advantages allow us to apply the Frame Algorithm in an environment that is difficult to control, namely the multi-cell network. Our results show that packet data networks can use very dense frequency reuse.

2 Model of System and Feedback

We address a cellular packet multiple-access network with feedback. At the end of each slot, the base station broadcasts a feedback message to all terminals in its cell containing its observation on the status of that slot, i.e., whether it was thought to be `idle', a destructive `collision', or a `successful slot'. The feedback channel is considered error free, so all terminals receive exactly the same information, but the base station may confuse idle slots and collisions and does not observe packets hidden below a capturing packet.

Packets have unity duration, each equal to the slot time. Slot $[t, t+1)$ is called slot t. Let us consider a particular cell, with ω_t the number of packets transmitted in slot t in that cell. By time $t+1$, the feedback channel reports which of the following three events occurred in slot t:

- $\{\varepsilon_t = 2\}$ there were transmissions of packets in slot t, but none of these transmission was successful. (`destructive collision'; `conflict').
- $\{\varepsilon_t = 1\}$ one transmission was successful in slot t. It is unknown whether other (unsuccessful) transmissions were also present (`capture').
- $\{\varepsilon_t = 0\}$ there were no transmissions in slot t ('idle')

We define the feedback error probabilities π_0 and π_1. Let

$$\pi_0 = \Pr\{\varepsilon_t = 2 | \omega_t = 0\} = 1 - \Pr\{\varepsilon_t = 0 | \omega_t = 0\}$$

so π_0 is the conditional probability that the base station of a considered cell decides that $\varepsilon_t = 2$ (conflict), given that there was no transmission in the considered cell.

Another error event is the base station's failure to correctly decode a data packet even though there was only a single transmission in time slot t ($\omega_t = 1$). The base station interprets this as a collision. It happens with conditional probability

$$\pi_1 = \Pr\{\varepsilon_t = 2 | \omega_t = 1\} = 1 - \Pr\{\varepsilon_t = 1 | \omega_t = 1\}$$

In such case all terminals regard this slot to be a collision, so a retransmission is to be performed according to the algorithm presented later.

The capture probability that one packet is successful, given the presence of $\omega_t = k$ competing packets is denoted as q_k, with

$$q_k = \Pr\{\varepsilon_t = 1 | \omega_t = k\} = 1 - Pr\{\varepsilon_t = 2 | \omega_t = k\}$$

and $q_1 = 1 - \pi_1$. Whenever capture occurs, the feedback always reports $\varepsilon_t = 1$. Moreover, the base station identifies which message was successful, by broadcasting the address id. of the terminal from which it originated. The $k - 1$ terminals that also transmitted a packet but were unsuccessful understand that their transmission failed. All other terminals remain unaware of whether the slot contain a single successful transmission or additional packets were obscured by the transmission.

In a practical radio system, packet signals may be too weak to detect their presence, in which case slot t, with $\omega_t > 0$ is interpreted as being idle. However, we assume that

$$\xi_k = \Pr\{\varepsilon_t = 0 | \omega_t = k\} = 0, \text{ for } k > 0$$

and $\pi_0 + \xi_0 = 1$. A terminal that transmitted a packet nonetheless received feedback reporting an idle slot ($\varepsilon_t = 0$). In such case it simply retransmits in the next slot.

3 Improved Frame Algorithm

The time axis is partitioned into frames $[nA, (n+1)A)$ with $n \in I_1$ and $A \in I_2$, where A is the frame length, expressed in number of time slots, n is the frame number and I_j are the integer numbers greater than j, $I_j = \{j, j+1, ...\}$. A new packet arriving in frame $(n-1)$ is not transmitted till time nA. If the initial transmission attempt in slot $[nA, nA+1)$ is successful the packet leaves the system, otherwise the packet changes its level L_t at each moment $t \in I_{nA+1}$ according to the algorithm rules, up to its success. The terminal computes, stores and updates the current value of L_t and the level of frames $l_t^{(k)}$ $k \geq n$ in a buffer.

The basic principle is that packets which arrive during one frame start their transmission in the next frame. All packets start with a transmission attempt at level $L_t = 0$, but are split by moving some packets from $L_t = 0$ to $L_t = 1$ every time when a collision occurs among the packets with $L_t = 0$. When no packets are left with $L_t = 0$, the packets with $L_t = 1$ move back to $L_t = 0$ and are retransmitted. This process continues until all packets of that frame are successfully retransmitted or till the end the frame (of A slots; at instants $t = mA$) occurs. If the frame ends before the collision is fully resolved, the untransmitted packets enter a backlog stack. Whenever the collision is resolved before the end the frame, unresolved collisions from previous frames are now resolved.

In summary, the Frame Algorithm resembles a two-level Stack Algorithm. Within one frame, two levels are used for rapid and effective collision resolution. To ensure stability, it gives LIFO type of packet access for new arrivals at frame level. The main advantage over other stack algorithms is its enhanced robustness against feedback errors, even if the error probability is very large.

Let us define $\Delta_t = 1$ if $t \in \{A, 2A, ...\}$, i.e., at frame starts, and $\Delta_t = 0$ otherwise. The formal rules for frame level changes are:

1. At moment nA the level $l_t^{(n)}$ starts to exist with initial value $l_{nA}^{(n)} = 0$.
2. If $\varepsilon_t = 0$, $l_t^{(n)} = 0$ then $l_t^{(n)}$ stops to exist.
3. If $\varepsilon_t = 0$, $l_t^{(n)} > 0$ then $l_{t+1}^{(n)} = l_t^{(n)} - 1$
4. If $\varepsilon_t = 1$, then $l_{t+1}^{(n)} = l_t^{(n)} + \Delta_{t+1}$.
5. If $\varepsilon_t = 2$, $l_t^{(n)} = 0$ then $l_{t+1}^{(n)} = 1 + \Delta_{t+1}$.
6. If $\varepsilon_t = 2$, $l_t^{(n)} > 0$, $\mathtt{min}_i\ l_t^{(i)} = 0$, then $l_{t+1}^{(n)} = l_t^{(n)} + 1 + \Delta_{t+1}$.
7. If $\varepsilon_t = 2$, $l_t^{(n)} > 0$, $\mathtt{min}_i\ l_t^{(i)} = 1$, then $l_{t+1}^{(n)} = l_t^{(n)} + \Delta_{t+1}$.

In rules 6 - 7, the `min` over i is taken over all frame levels which exist at moment t. If at any moment t two frame levels $l_t^{(i)}$ and $l_t^{(j)}$ exist with $>j$ then $l_t^{(i)} < l_t^{(j)}$.

The rules for packet level L_t changes are

1) $L_{nA} = 0$
2) If $\varepsilon_t = 1$, $l_t^{(n)} \leq 1$, $L_t = 0$, the packet that receives an acknowledgment leaves the system. Other packets remain at their position.
3) If $l_t^{(n)} > 1$, $L_{t+1} = L_t$
4) If $\varepsilon_t = 0$, $l_t^{(n)} \leq 1$, $L_t = 1$, then $L_{t+1} = 0$
5) If $\varepsilon_t = 1$, $l_t^{(n)} \leq 1$, $L_t = 1$, then $L_{t+1} = 1$
6) If $\varepsilon_t = 2$, $l_t^{(n)} \leq 0$, $L_t = 1$, then with probability ½, $L_{t+1} = 0$ and with probability ½, $L_{t-1} = 1$. (Colliding packets split)
7) If $\varepsilon_t = 2$, $l_t^{(n)} \leq 1$, $L_t = 1$, then $L_{t+1} = 1$

For any moment t, $L_t \leq l_t$. Each terminal calculates the level of its own packet L_t which arrived in frame $n - 1$ and all frames $l_t^{(k)}$ with $k \geq n$. The algorithm is 'limited-sensing': terminal with a packet arriving during frame $n - 1$ does not need to know the feedback ε_t for $t < nA$. However the algorithm requires that terminals know the instants mA of the beginning of each frame.

4 Brief Summary of the Analysis

The total flow of new packets is discrete-time Poissonian with intensity λ packets per slot per cell. Let, within some frame $(n - 1)$ at time $t^{(0)}$ uniformly distributed over the frame length A, a new packet arrive in addition to the Poisson input flow. This packet is called the 'test' packet. Let $t^{(1)}$ denote the time of the beginning of the slot in which the test packet achieves its successful transmission and then leaves the system. The random variable δ with $\delta = t^{(1)} - t^{(0)}$ is called the packet delay. For the considered algorithm the probability distribution of δ does not depend on n. The average packet delay is defined as the expectation $D = E\delta$. The throughput is defined as $R = \sup\{\lambda: E\delta < \infty\}$

The find the throughput and delay, we need to denote a being the arrival rate per frame, ($a = \lambda A$), ρ_n being the number of slots in session n form its beginning till the slot in which the test packet succeeds and then leaves the system, and τ_n being the length of session n. Also,

$$W_1 = \sum_{i=0}^{A-1} (A - i - 1) \Pr(\rho_n = i)$$

and

$$W_2 = \sum_{i=0}^{A-1} (1-i) \Pr(\rho_n = i)$$

In [2], an algorithm with the above structure was proposed but with a different conflict resolution procedure, suitable for errorless feedback ($\pi_0 = \pi_1 = 0$) and without capture ($q_k = 0$ for $k > 1$). For average delay, [2] gives the bounds

$$\frac{E\tau_n(E\rho_n - A + 1 + W_1)}{A - E\tau_n} \le D - \frac{A}{2} - E\rho_n \le \frac{E\tau_n(E\rho_n - 1 + W_2)}{A - E\tau_n}$$

where $E\rho_n$, $E\tau_n$, W_1 and W_2 do not depend on n but depend on a. Despite the more complicated situation in this paper, the above bounds also holds for our case, but the values of $E\rho_n$, $E\tau_n$, W_1 and W_2 have different expressions than in [2]. For $A = 2$ it is easy to see that the lower bound and upper bounds coincide since $W_1 = W_2 = \Pr(\rho_n = 0)$ [2].

Similar to [2], it can be shown that $E\tau_n < \infty$, $E\rho_n < \infty$ when $A < \infty$, $\pi_0 < 1$, $\pi_1 < 1$. Hence, it follows from the bounds that the algorithm throughput is the solution of the equation $E\,\tau_n(R\,A) = A$, where $E\,\tau_n(R\,A)$ is the expectation of τ_n dependent on $a = \lambda A$ at $\lambda = R$.

A Throughput Calculation

To expressed R in λ, π_0, π_1 and q_k, we initially address $E\tau_n$. We have

$$E\tau_n = \sum_{i=0}^{\infty} T_i \frac{a^i}{i!} e^{-a}$$

where T_i is the conditional expectation of the length of session n, given i new packets arrived in session (n - 1), $i \in I_0$. In this version we omit the derivation from which we are able to calculate the root R numerically.

B Average Delay Calculation

We will use the bounds to find the average packet delay. Initially we compute $E\rho_n$ from

$$E\rho_n = \sum_{k=0}^{\infty} H_k \frac{a^k}{k!} e^{-a}$$

where H_k is the conditional expectation of ρ_n, given the session n begins with a conflict of multiplicity (k + 1), the test packet including. Using the derivation of T_i, H_k, $E\tau_n$ and $E\rho_n$ omitted here, we can bound the average packet delay D.

5 Radio Channel Characterization [3,4]

We address Rayleigh fading wireless channels. The instantaneous power p_i received from the i-th user is exponentially distributed with mean q_i. A commonly used path loss model used for analysis of generic radio systems is $q_i = r_i^{-b}$ where $b \approx 4$. Each base station receives signals from a cell with normalized distances ranging between $0 < r_i < 1$. This is a circular approximation of a typical hexagonal cell with size $\sqrt{(2\pi)/3^{3/4}}$. We use C to denote the frequency reuse cluster size. Note that C has an effect both on the amount of bandwidth available per cell, and on the amount of interference seen from other cells. The transmission time of a packet is proportional to C. Hence, for a given arrival rate expressed in packets per *second*, the expected number of packets per *slot* is proportional to C.

We assume that a message is received successfully iff the signal-to-interference ratio exceeds z, and we use z = 4 (6 dB). Assuming constant received power during a packet transmission time, the probability that the signal power p_0 of the packet with index 0 sufficiently exceeds the joint interference p_t is

$$\Pr(p_0 > z p_t) = \int_{0-}^{\infty} f_{p_t}(x) \int_{zx}^{\infty} f_{p_0}(y) dy dx$$

where we insert the appropriate pdf f_{p0} of the received power due to Rayleigh fading and path loss, associated with a random terminal location and the pdf f_{pt}. Note that p_t is the sum of multiple individual signals, i.e., $p_t = p_1 + p_2 + .. \; p_{n+k}$, where $\omega_t = k$ and $\chi_t = n$ are the number of signals transmitted in slot t within and outside the considered cell, resp. For capture probabilities conditional on q_0, we get [3]

$$\Pr\left(p_0 > z\,p_t \middle| q_0 = r_0^{-b}\right) = L\left\{p_t;\; s = z r_0^b\right\} = E\,e^{-z p_t r_0^b}$$

where $L\{p;\, s\}$ is the Laplace transform of the pdf of p at s. For independent fading and independent terminal locations, the pdf of the joint interference power is the convolution of the pdf of individual interference powers. Its Laplace Transform is the product of the transforms of individual interfering pdfs. Moreover, the image of the sum Y ($Y = X_0 + X_1 + ...$) of a Poisson-distributed random number of i.i.d. random variables equals

$$L(Y; s) = \exp\left(-\mu\left[L(X_i; s) - 1\right]\right)$$

where μ is the arrival rate.

Transmissions in neighboring cells are approximated to be independent, i.e., we simplify the interaction amoung cells. The transmission rate per cell $\mu = E\omega_t$ is obtained from λ plus the expected number of retransmissions. We address a spatially uniform system with identical arrival rates in all cells. From numerical experiments it appears that for arrival rates λ less than $2R/3$, relatively few retransmissions occur ($\mu \approx \lambda$). At larger λ, many retransmission attempts will occur and μ typically is close to one. We will assume a stepwise transition at $\lambda = 2R/3$. In some curves, this resulted in a small visible jump in delay.

A. Interference Model for $C = 1$

For a single interfering terminal i at a random location in a circular band $r_1 < r < r_2$, the Laplace transform of the pdf of received power can be computed as

$$L(p_i; s) = 1 - \frac{\sqrt{s}}{r_2^2 - r_1^2} \arctan \frac{\sqrt{s}\left(r_2^2 - r_1^2\right)}{s + r_2^2 r_1^2}$$

if we take $b = 4$. For instance, for an interferer known to be in the considered cell ($r_1 = 0$; $r_2 = 1$)

$$L(p_i; s) = 1 - \sqrt{s} \arctan \frac{1}{\sqrt{s}}$$

If $C = 1$, the interference from outside the cell is a contiguous, infinitely extended spatial arrival process, we insert $r_1 = 1$; $r_2 >> 1$. For a single interferer. We are interested in the case of a known number of m interfering signals within the considered cell ($0 < r < 1$) and a random number of n signals coming from a large area outside the cell $1 < r < r_2$. We approximate n to be a Poisson random number, independent of k, with mean $\mathrm{E}n = \mu(r_2^2 - 1)$. Considering signal powers to be i.i.d. random variables, the Laplace image of joint signal power, given $\omega_t = m$ packets in the cell is, after taking the limit for $r_2 \to \infty$,

$$L(p_t; s) = \left[1 - \sqrt{s} \arctan \frac{1}{\sqrt{s}}\right]^m \exp\left(-\sqrt{s}\mu \arctan\sqrt{s}\right)$$

To find the probability of capture for one packet (out of k) with known location r_0 we insert $s = z\, r_0^4$ and replace m by $k - 1$. We average the result over the location of the reference packet to find

$$q_k = k \int_0^1 L\left(p_t; s = z r_o^4 \middle| m = k - 1\right) 2 r_0 dr_0$$

B. Interference Model for large C

If C is large, only the first tier of co-channel cells significantly contribute interference. We approximate the received local mean power to be constant, i.e., all mobiles are modeled to be close to the cell center. The distance between two co-channel cells, is $\sqrt{(3C)} R_{hex}$ with R_{hex} the size of the hexagonal cell. A circular cell with unity radius $R = 1$ has the same surface area as a hexagonal cell with size $R_{hex} = \sqrt{(2\pi)}/3^{3/4}$. So $q_i = 3/(4\pi^2 C^2)$. Using the Laplace image of interference power, given the presence of m interferers, the probability that a particular packet from distance r_0 captures the base station is

$$L(p_t; s = z r_0^4) =$$

$$\left[1 - \sqrt{z}\, r_0^2 \arctan \frac{1}{\sqrt{z}\, r_0^2}\right]^m \exp\left(-6\mu \frac{3 z r_0^4}{4\pi^2 C^2 + 3 z r_0^4}\right)$$

6 Slot Content Estimation

If the base station cannot successfully receive a packet, it establishes whether the slot was 'idle' or occupied by colliding packets. It typically uses a power measurement and a threshold device to distinguish between an idle slot and a collision. If the received power exceeds P_{thr}, the base station decides that the slot contains a destructive collision. Signals blown over from other cells may obscure this observation and cause errors in the feedback. We can find π_0 from the distribution of p_t. Its Laplace transform, given $\omega_k = 0$ is, for $C = 1$

$$L(p_t; s) = \exp\left(-\sqrt{s}\mu \arctan\sqrt{s}\right)$$

and for large C, considering one tier of 6 co-channel base stations

$$L(p_t; s) = \exp\left(-6\mu \frac{3s}{4\pi^2 C^2 + 3s}\right)$$

Inverse transformation is needed to compute π_0. To find a crude but simple approximation for π_0, one can insert $s = 1/P_{thr}$.

7 Computational Results

Fig. 1 presents capture probabilities versus the number of signals involved under various conditions of interfering traffic in co-channel cells. For cluster size $C = 3$ (solid curve "1"), the amount of traffic in other cells has practically no effect. For $C = 1$, the effect of interference is severe.

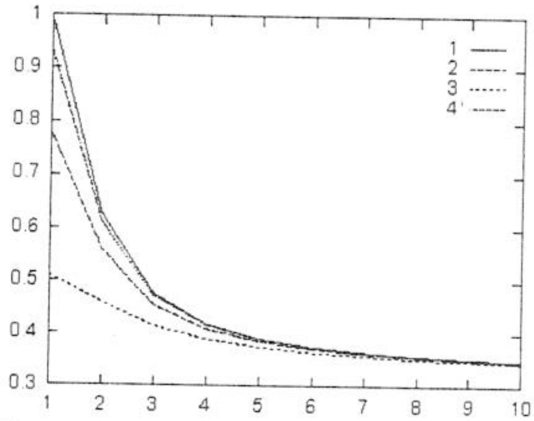

Fig. 1 Probability q_k that one out of k signals in the cell captures the base station. Curve "1": $\mu = 0$, $C = 1$, "2": $\mu = 0.3$, $C = 1$, "3": $\mu = 1$, $C = 1$, "1": $\mu = 1$, $C = 3$. Capture ratio: $z = 4$.

Feedback errors in Fig. 2 illustrate the effect of interference from other cells on the slot content estimation. The probability that a collision slot is interpreted as being idle rapidly decreases with the number of interferers k. One can make $\xi_k < 0.05$ (taking $P_{thr} = 1$) if one accepts that $\pi_0 = 0.55$. For larger C ($C = 3, 4 \ldots$), π_0 becomes close to zero.

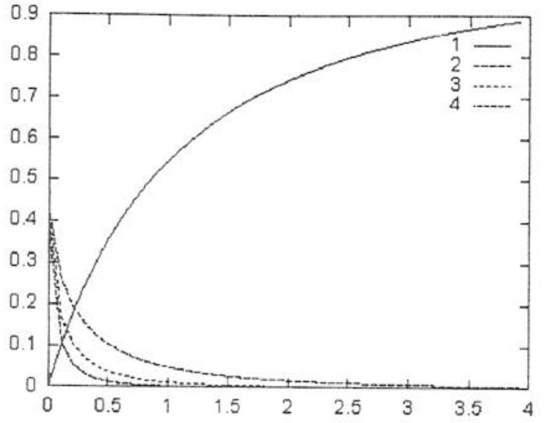

Fig. 2 Probability π_0 ("1") that an empty slot is erroneously interpreted as a collision for C = 1 and Probability ξ_k ("2":k=1; "3":k=2; "4":k=3;) that occupied slots is erroneously understood to be idle, versus the detection threshold P_{thr}.

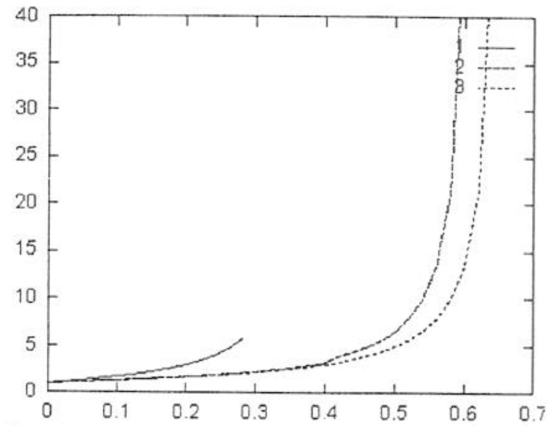

Fig. 4 Delay versus arrival rate λ per slot. Frame length A = 2. Curve "1":C=1; "2":C=3; "3":C=4. (Note that the arrival rate per *second* is proportional to λ/C.)

Fig. 3 compares throughput for different reuse factors C. A C-cell reuse system can use only the fraction $1/C$ of all radio resources for each cell. For large C, this typically results in a low throughput and large delays. It appears that this effect is more significant than the effect that with large C the interference from other cells reduces. However, if C = 1, strong signals arrive from all neighboring cells, which causes interference, affects π_0 and the delay performance of the algorithm.

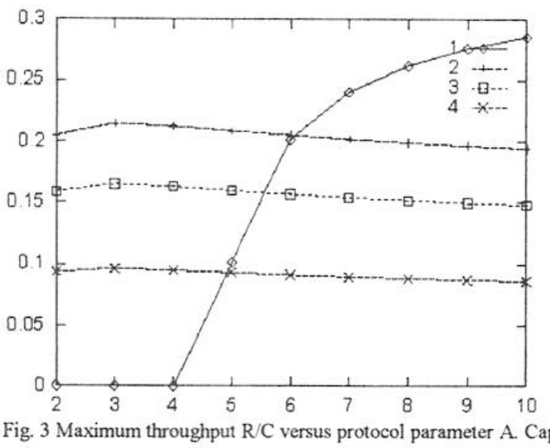

Fig. 3 Maximum throughput R/C versus protocol parameter A. Capture ratio z = 4. Detection threshold P_{thr} = 1, μ = 1
Curve "1":C=1; "2":C=3; "3":C=4; "4":C=7.

Nonetheless, in Fig. 3, an optimum choice for the cluster size seems $C = 1$ with large A. A large frame size makes the algorithm itself less efficient, but it appears advantageous to combat cross-cell interference. However, for large A, our model for μ becomes less accurate (optimistically low). For $C = 1$, the curve is aborted around $\lambda = 2R/3$, where μ is about unity and the delay increases steeply. In Fig. 4, the delay is given versus λ. Accounting for the reuse, the arrival rate per second is proportional to λ/C, so we see that small C gives best performance.

8 Conclusions

We have proposed a random access scheme that allows very dense frequency reuse (small cluster sizes C) in wireless multi-media networks. Theoretically, it achieves spectrum efficiencies of about 0.2 .. 0.3 bit/s/Hz/cells, which is high compared to existing cellular telephone systems, carrying non-bursty traffic. The effect of intercell interference is effectively resolved by our collision resolution scheme, possibly except for the case $C = 1$, where instability may occur among neighboring cells. Some further investigation into interaction of neighboring cell (at $C = 1$) is needed, particularly on how transmissions in one cell cause 'back-off' in other cells through influencing the feedback estimates (see π_0). Otherwise $C = 3$ would be a good solution.

We believe that these results are relevant in the design of future packet-switched wireless networks carrying bursty multi-media traffic.

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